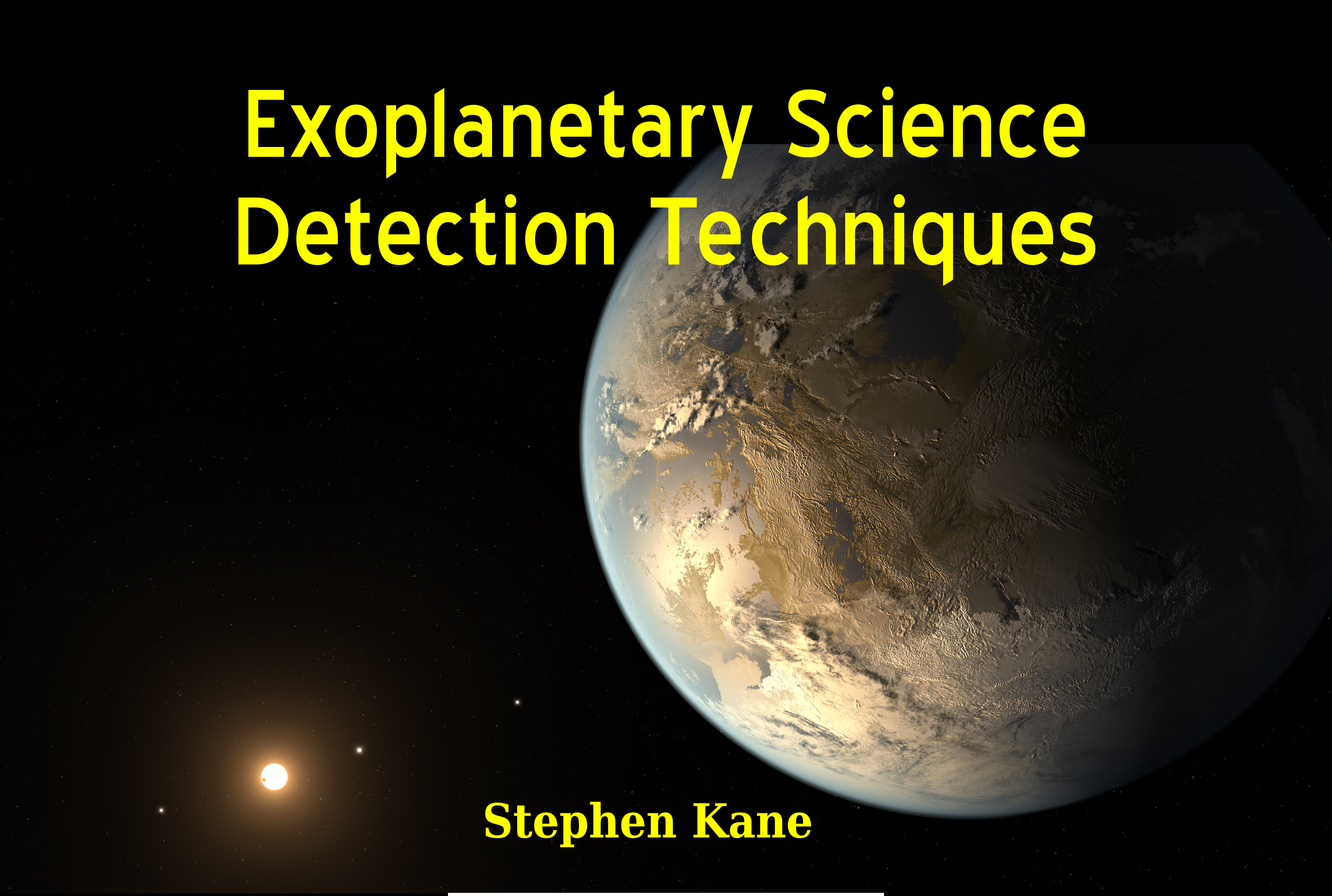

Exoplanetary Science Detection Techniques

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Topics

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- Lecture 2 – Stars I
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- Lecture 4 – The Solar System
- Lecture 5 – Exoplanet History I
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Johannes Kepler

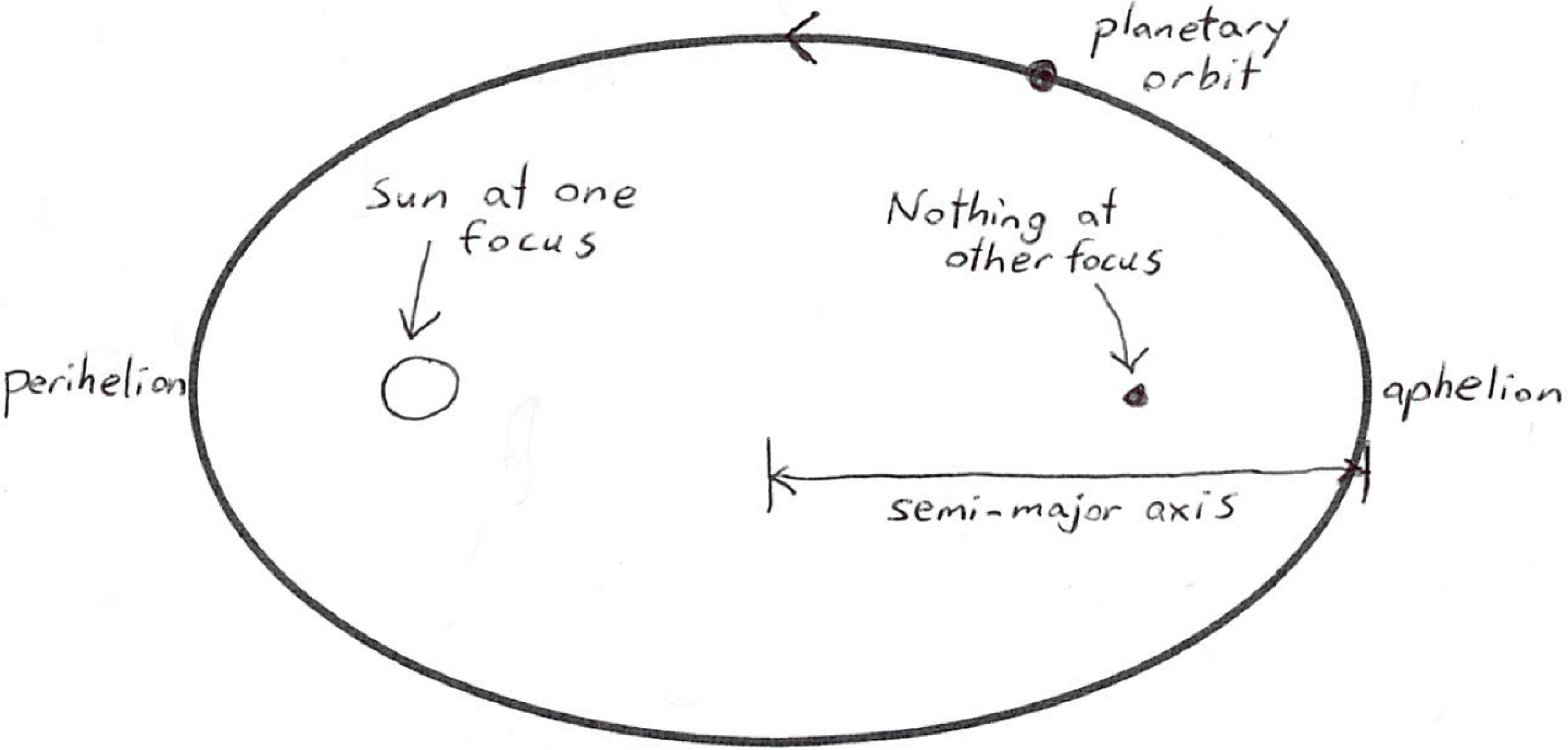
- German astronomer & mathematician (1571-1630)

- Was an assistant to the astronomer Tycho Brahe in Prague.

- Kepler used Brahe’s accurate positions of the planets to derive the laws of planetary motion.

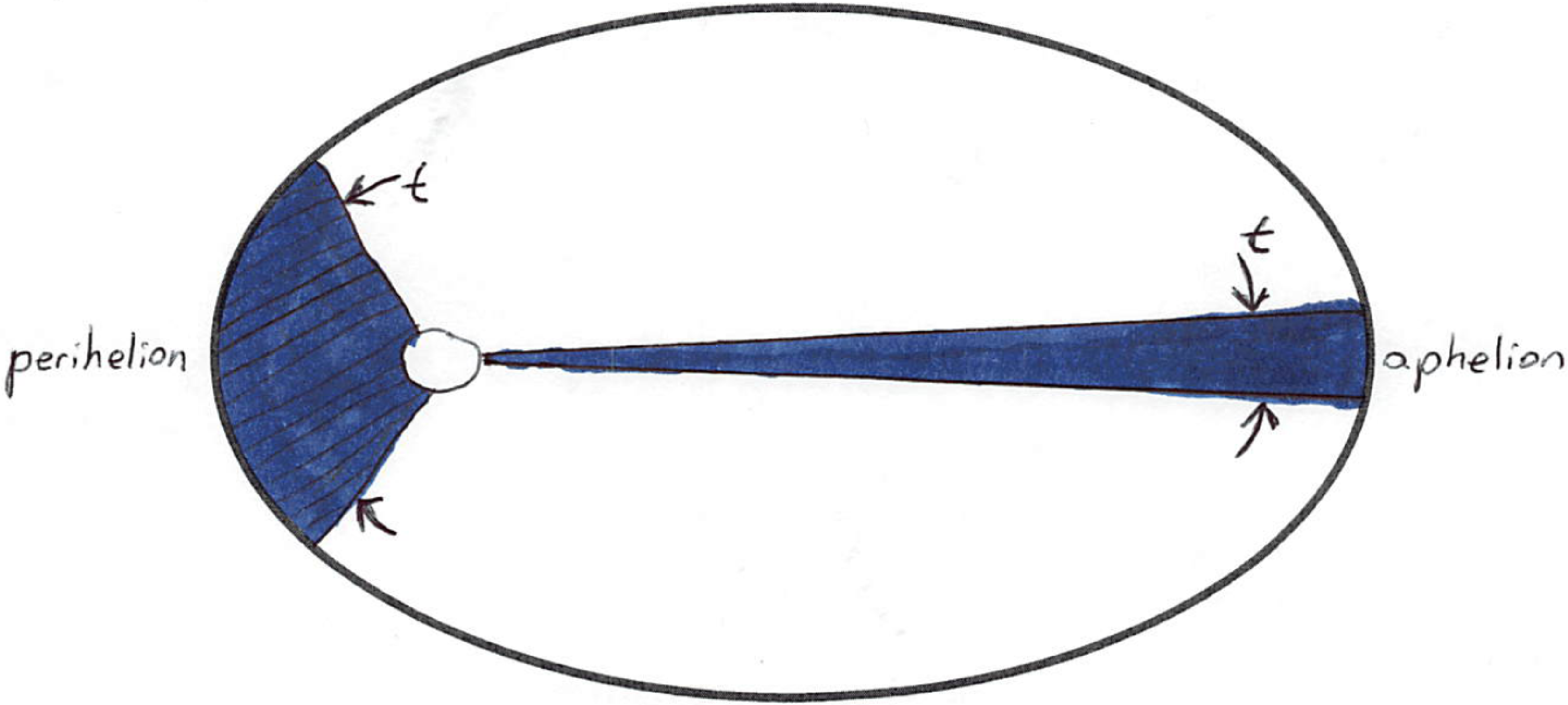
Kepler's laws of planetary motion

Law #1: The orbit of each planet around the Sun is an ellipse with the Sun at one focus.



Kepler's laws of planetary motion

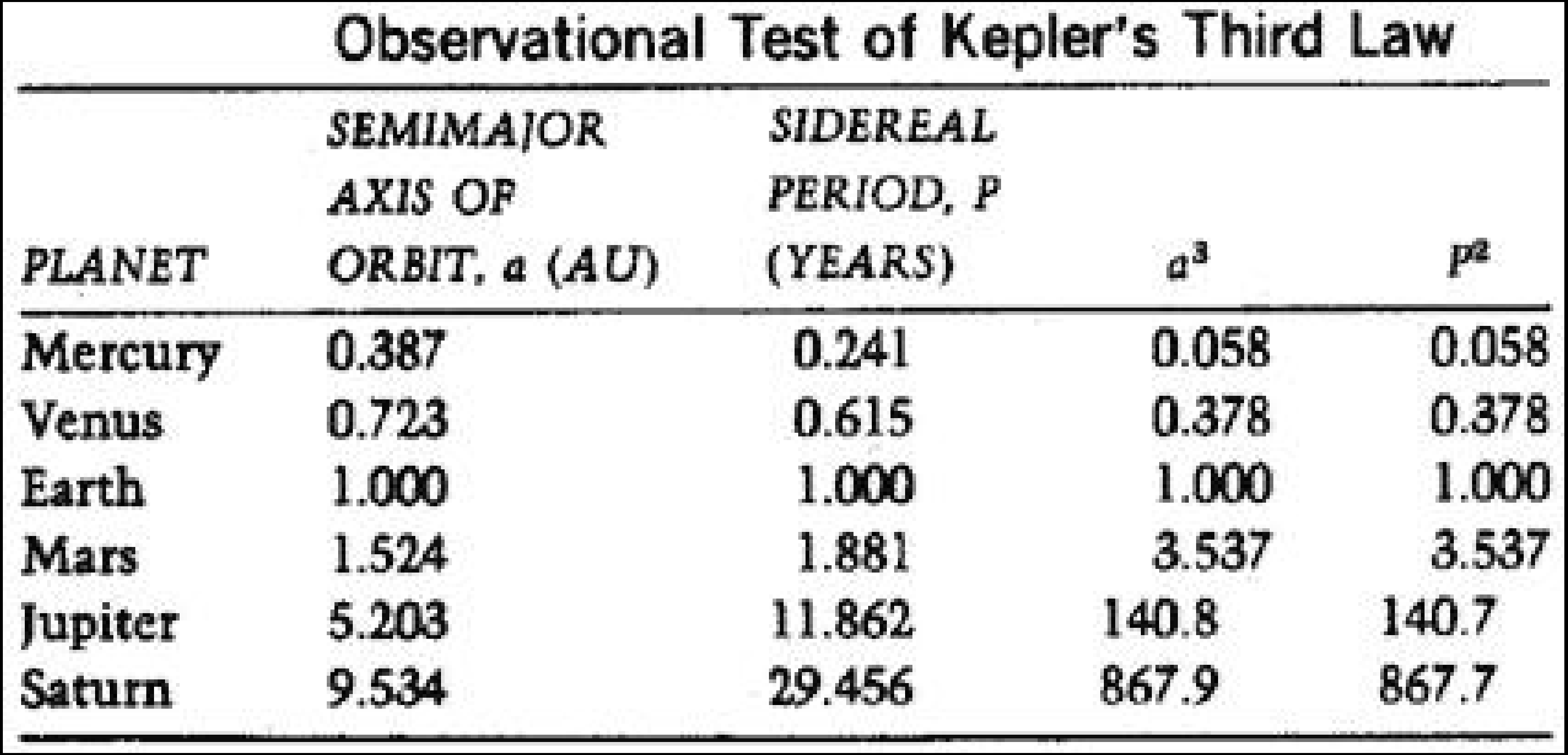
Law #2: As a planet moves around its orbit, it sweeps out equal areas in equal times.



Kepler’s laws of planetary motion

Law #3: More distant planets orbit at slower average speeds, obeying the relationship:

$$P^2 = a^3$$

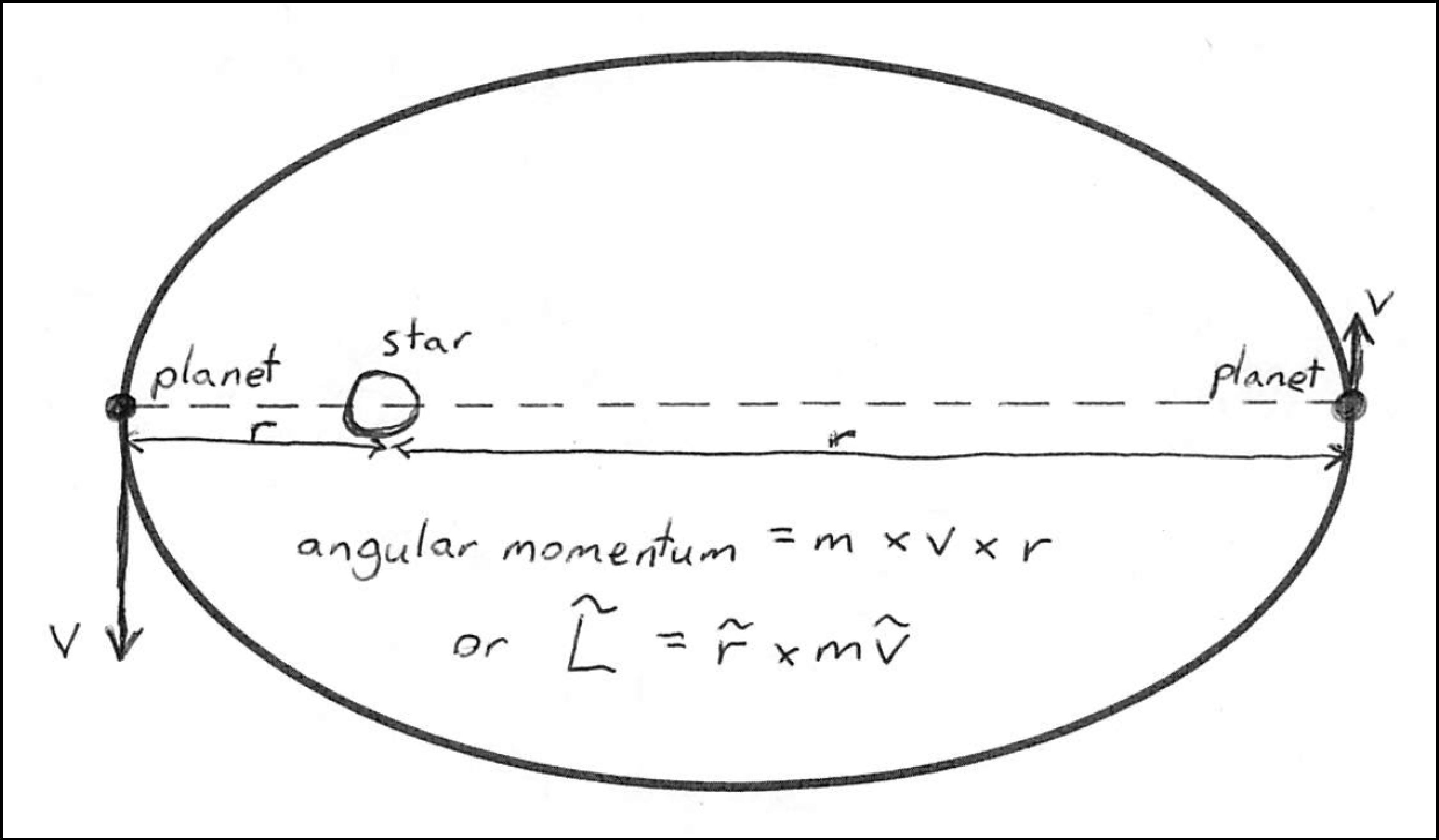


Observational Test of Kepler's Third Law

PLANET	*SEMIMAJOR AXIS OF ORBIT, a (AU)*	*SIDEREAL PERIOD, P (YEARS)*	a^3	P^2
Mercury	0.387	0.241	0.058	0.058
Venus	0.723	0.615	0.378	0.378
Earth	1.000	1.000	1.000	1.000
Mars	1.524	1.881	3.537	3.537
Jupiter	5.203	11.862	140.8	140.7
Saturn	9.534	29.456	867.9	867.7

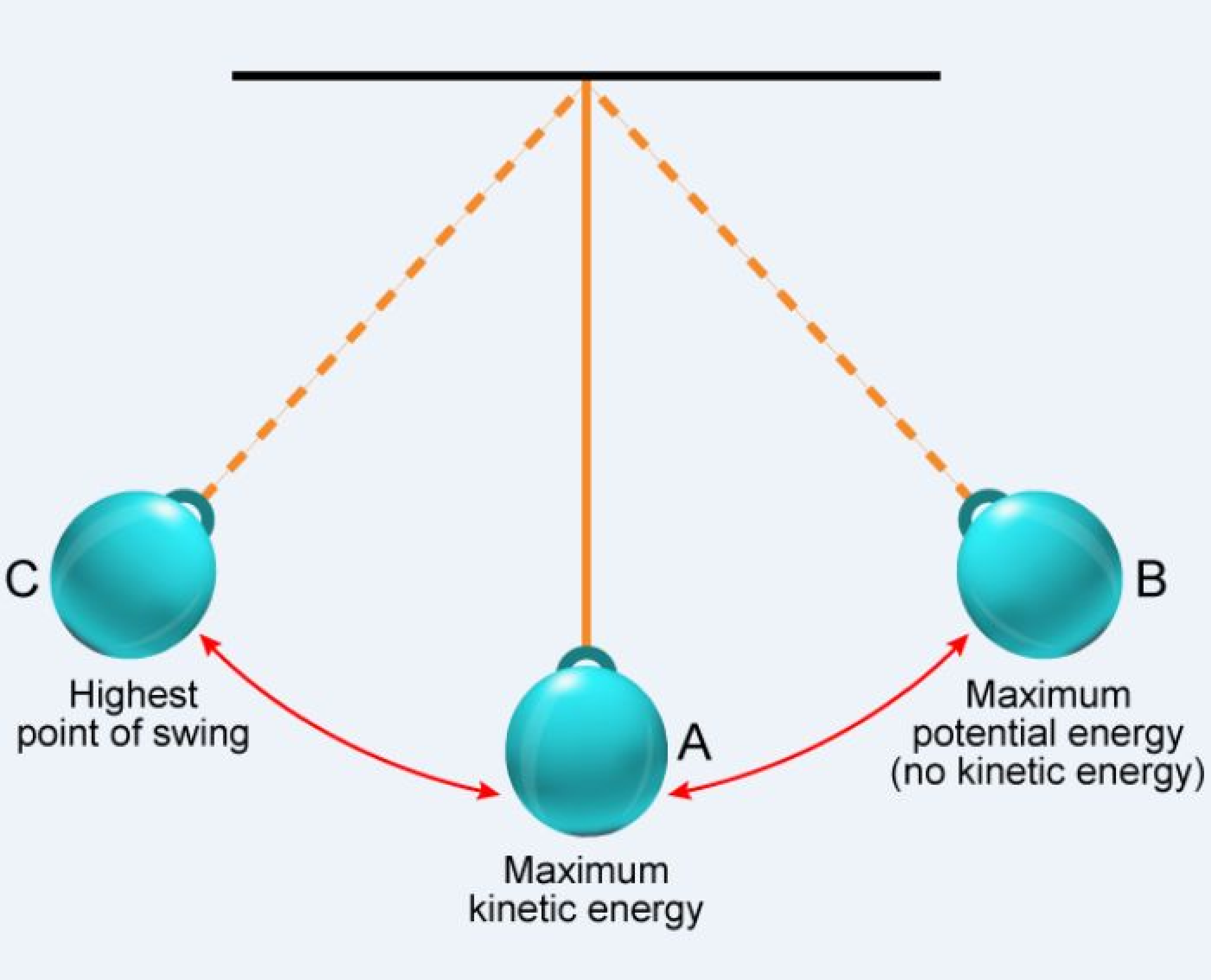
Kepler’s laws of planetary motion

Conservation of angular momentum: total angular momentum can never change (Kepler’s second law).



Kepler's laws of planetary motion

Conservation of energy: total orbital energy, sum of kinetic energy and potential energy, remains the same.



Newton's version of Kepler's Third Law

Kepler's third law is specific for the case of a one solar mass central body, as is the case for the solar system. Isaac Newton applied his law of universal gravitation to derive a generalized form of Kepler's third law:

$$P^2 = \frac{4\pi^2}{G(M_1 + M_2)} a^3$$

This is a powerful equation that can used to determine the mass of many orbiting objects. Usually $M_1 >> M_2$.

Newton’s version of Kepler’s Third Law

Example 1: How can we measure the mass of Jupiter through observations of the Galilean moons?

$$P^2 = \frac{4\pi^2}{G(M_1 + M_2)}a^3$$

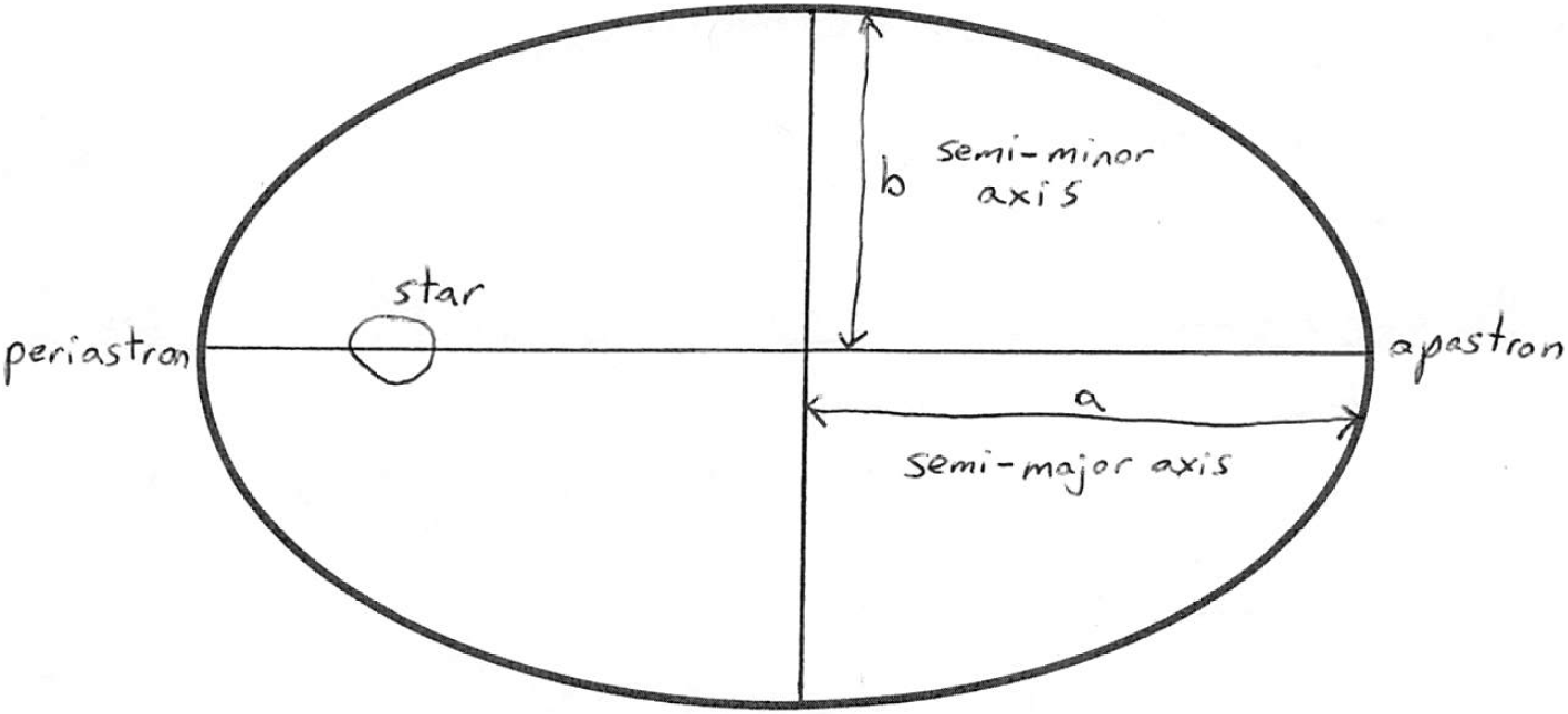
Newton's version of Kepler's Third Law

Example 2: The star HD 104985 is a ~1 solar mass star. A planet orbiting the star has an orbital period of 200 days and a mass of 4.9 Jupiter masses. What is the semi-major axis of the planet?

$$P^2 = \frac{4\pi^2}{G(M_1 + M_2)} a^3$$

Keplerian orbits

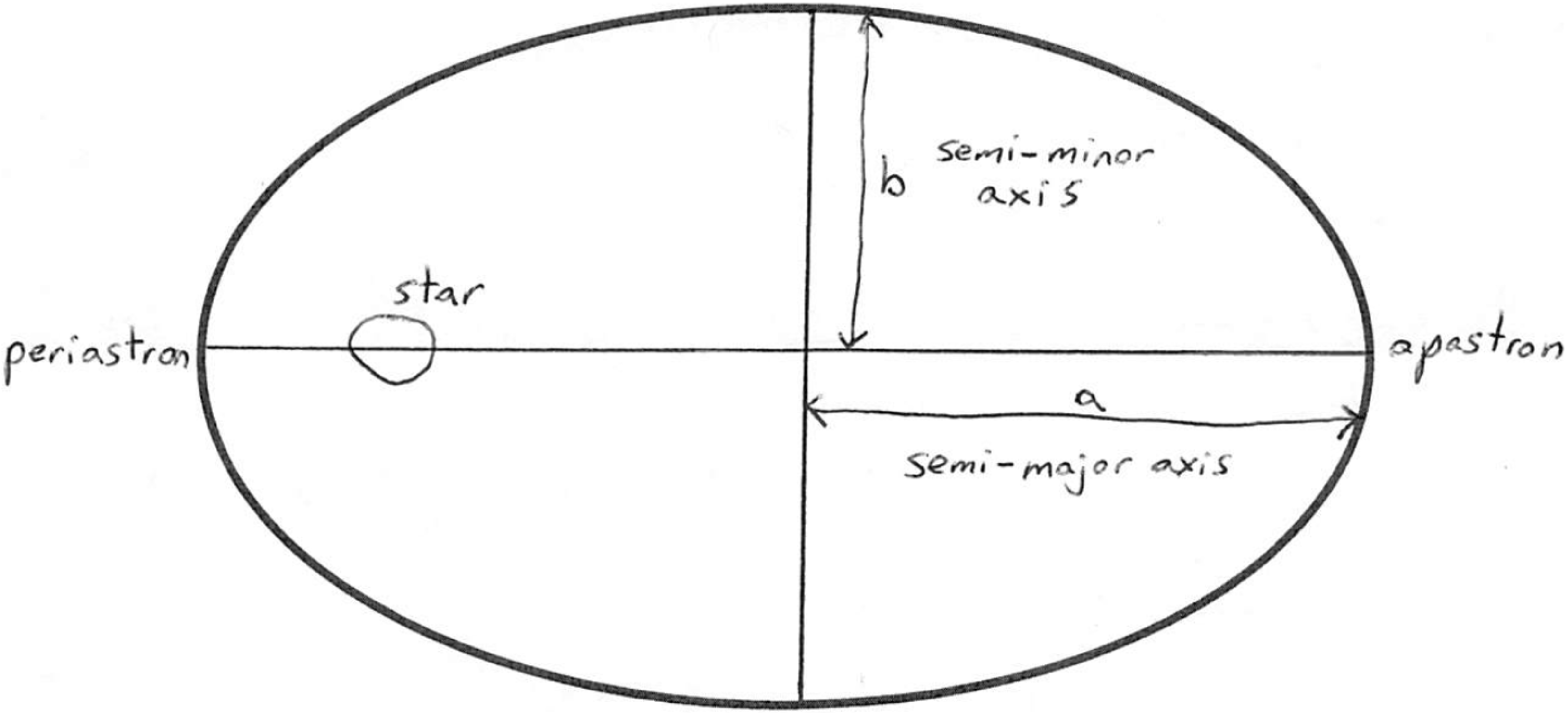
The objective for calculating a Keplerian orbit is to determine polar coordinates for the planet position at any time *t*. That means calculating a star-planet separation and reference angle.



Keplerian orbits

Geometry of an elliptical orbit. We define *e* as the eccentricity. For a bound orbit, eccentricity varies bteween 0 and 1.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad b^2 = a^2(1 - e^2) \Rightarrow e = \sqrt{1 - \left(\frac{b}{a}\right)^2}$$



Keplerian orbits

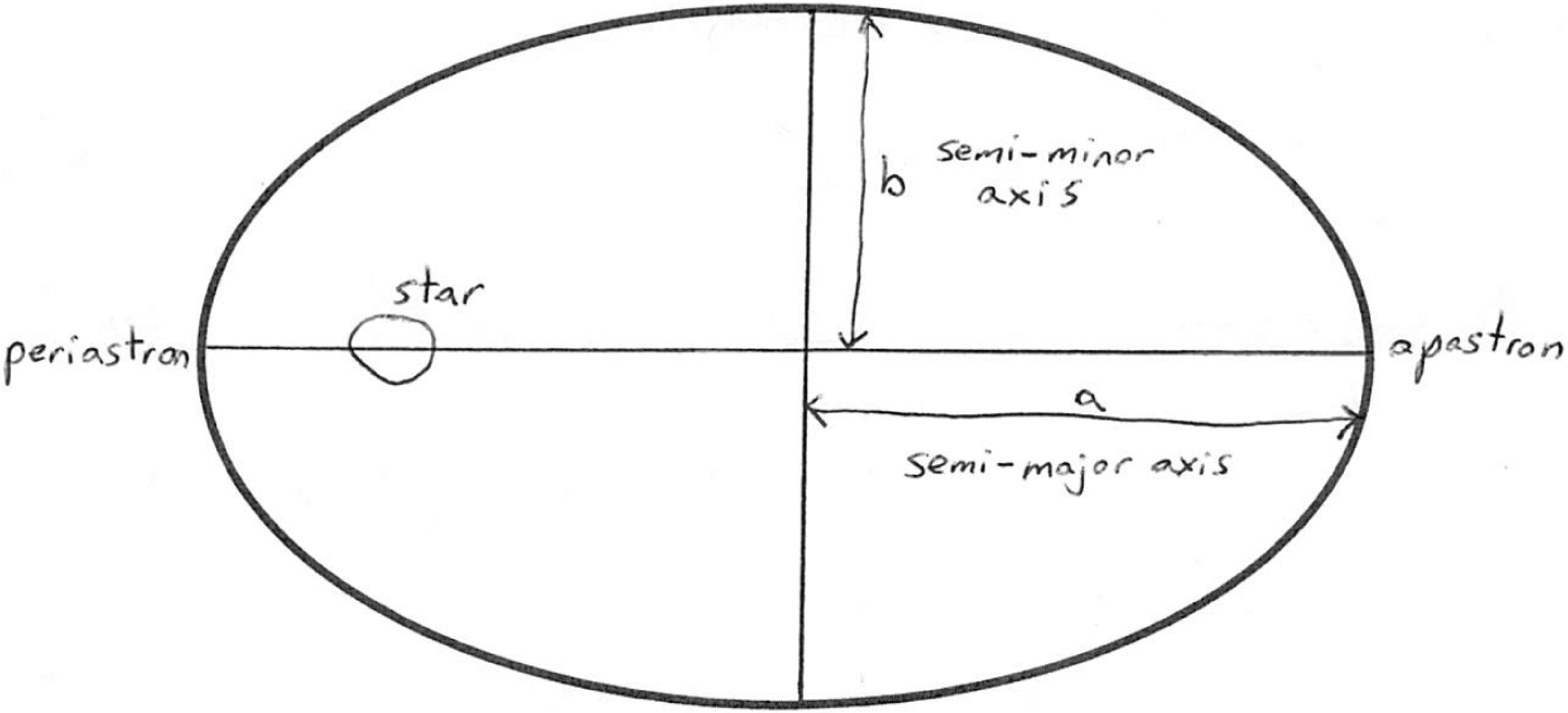
Let *r* be the star-planet separation.

At periastron: $r = a(1-e)$

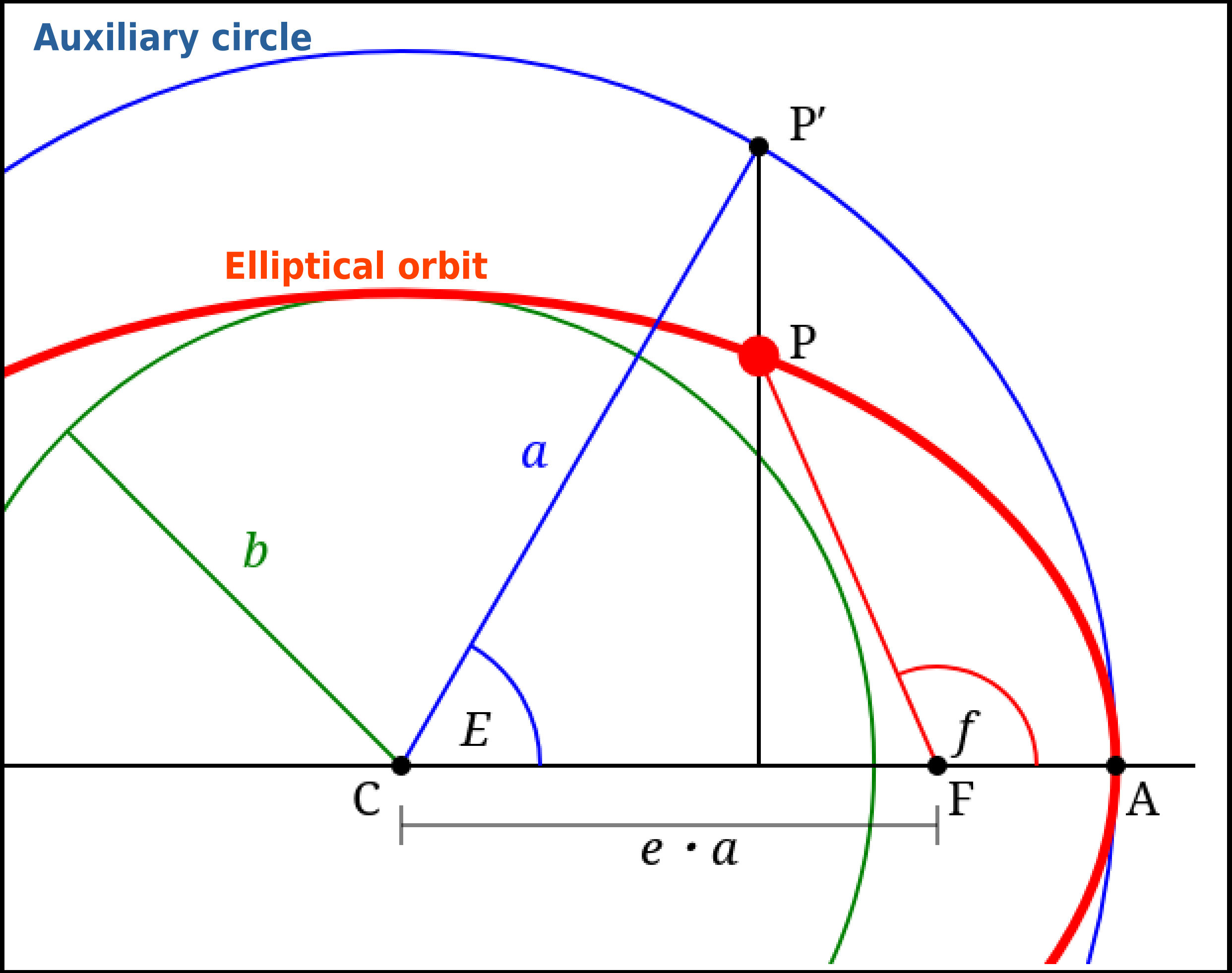
At apastron: $r = a(1+e)$

Periastron is sometimes called “pericenter” or “periapsis”.

Similarly for apastron.

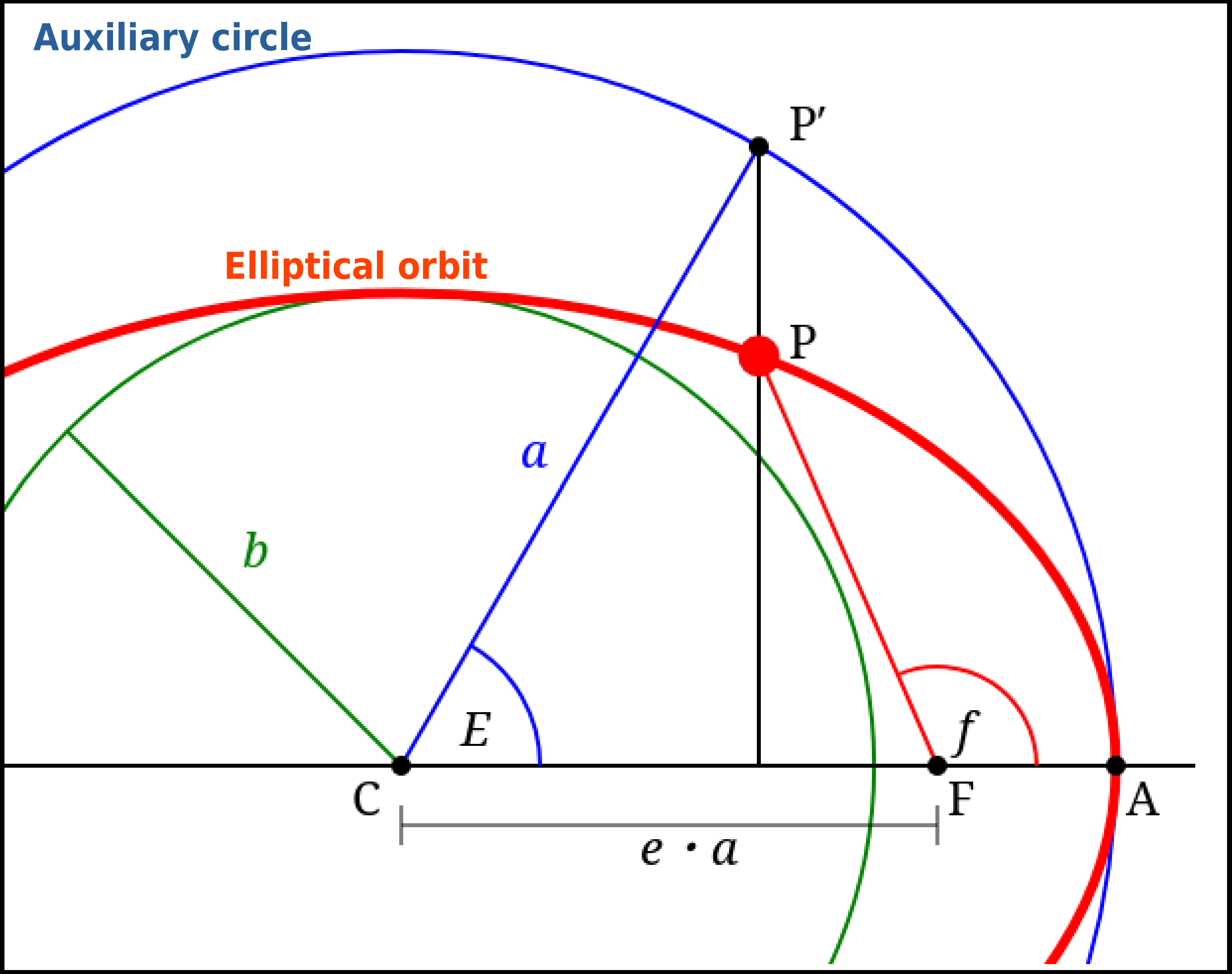


Time dependence of orbit



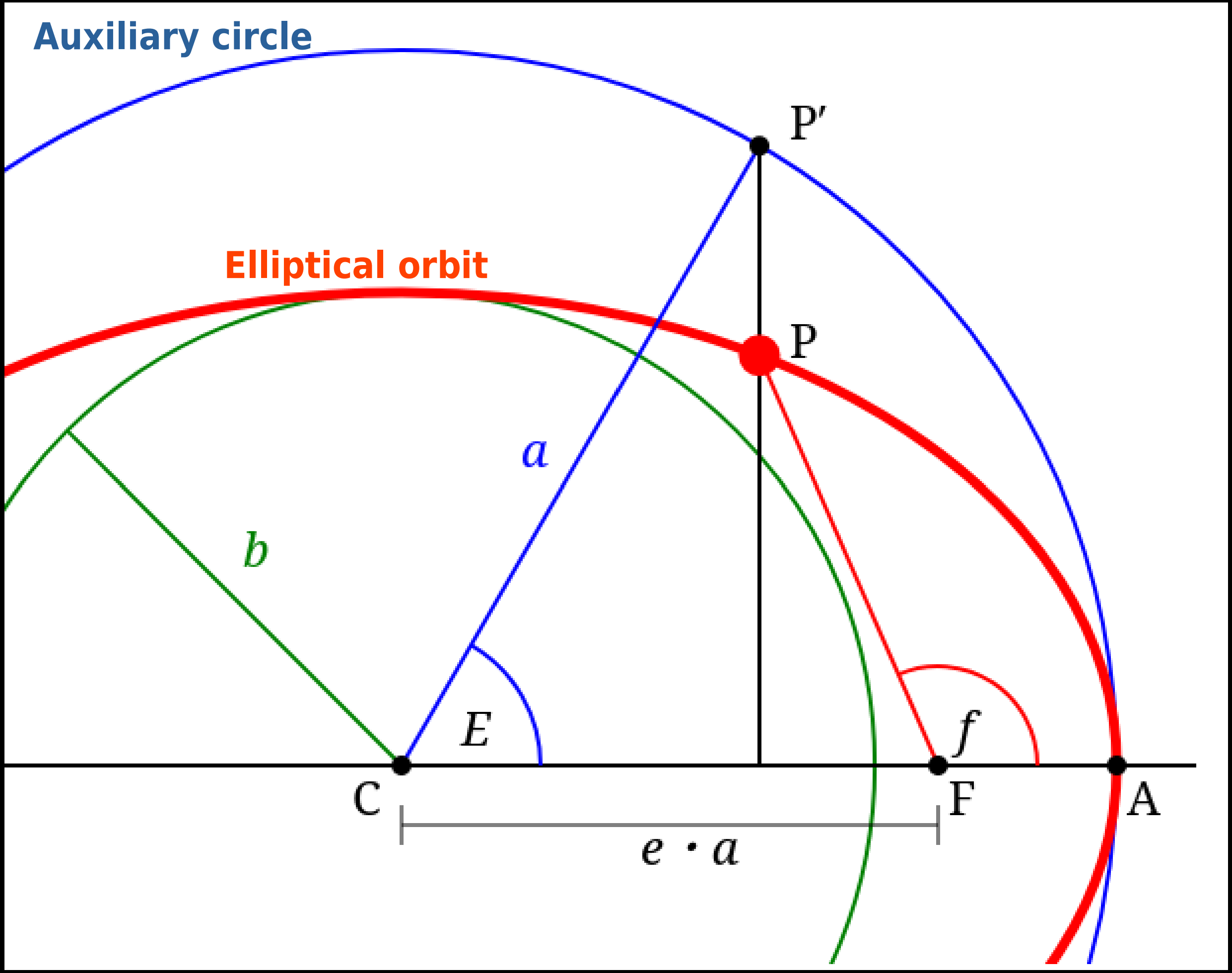
There are three main time dependent angles in an elliptical orbit, called anomalies.

Time dependence of orbit



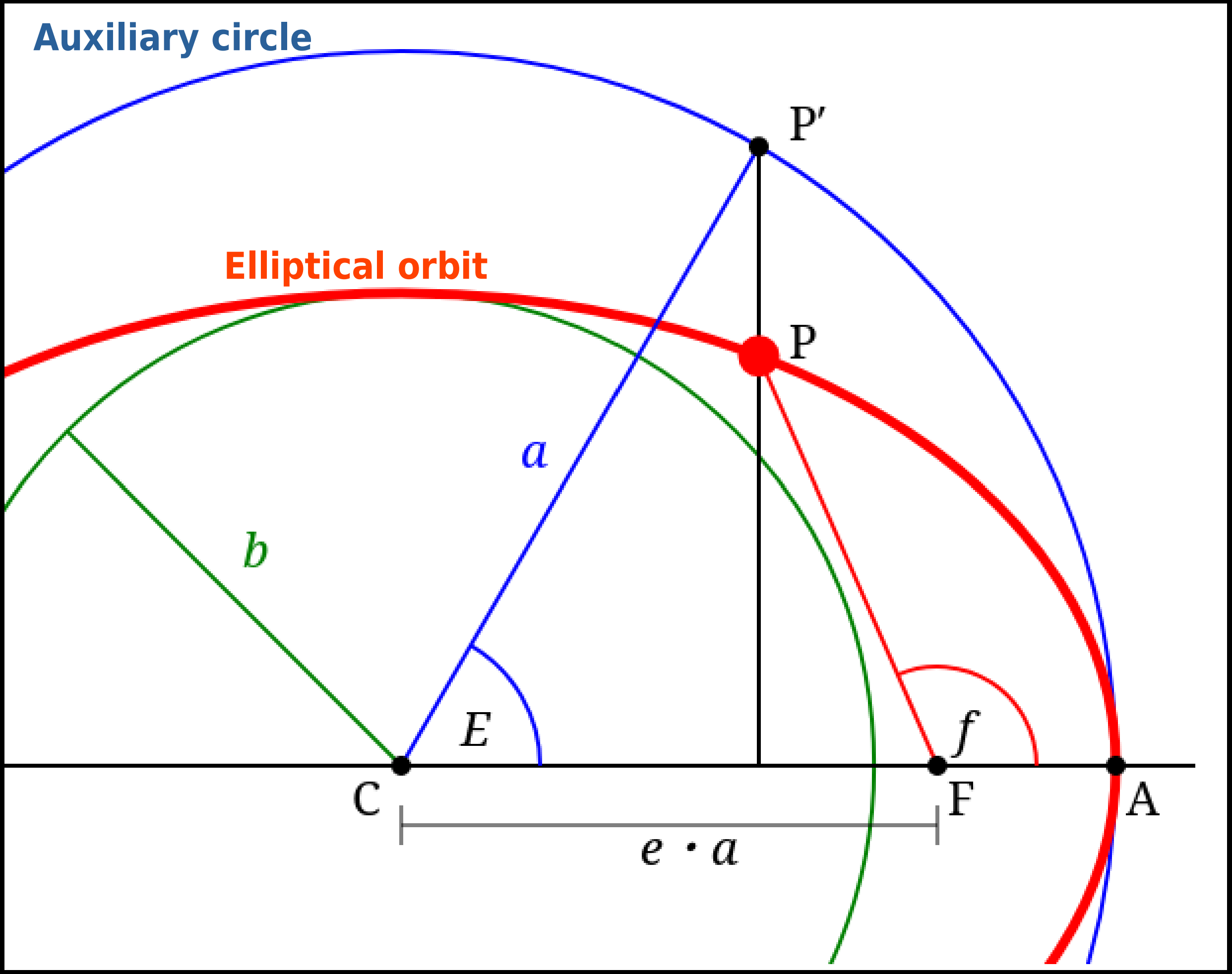
The true anomaly, *f(t)*, is the angle between the periastron direction and the planet position measured from the ellipse barycenter. This is shown above as angle PFA.

Time dependence of orbit



The eccentric anomaly, *E(t)*, is the angle between the periastron direction and the planet position projected onto the auxiliary circle. This is shown above as angle P’CA.

Time dependence of orbit



The mean anomaly, *M(t)*, represents a fictitious mean motion around the orbit. This is not represented on the diagram.

Time dependence of orbit

The mean anomaly, *M(t)*, represents a fictitious mean motion around the orbit, and is thus given by

$$M(t) = \frac{2\pi}{P}(t - t_p)$$

Where n ≡ 2π/P is the mean angular rate of motion, and t_p is the time at periastron passage.
***M(t)* and *E(t)* are related by the following expression:**

$$M(t) = E(t) - e \sin E(t)$$

This is referred to as Kepler's equation. This is a transcendental equation, which means it has no analytical solution and must be solved numerically (or graphically).
e.g., *x* = *cos(x)*.

Time dependence of orbit

By calculated *M(t)* and solving Kepler's equation for *E(t)*, we can find the true anomaly from the following relation:

$$\cos f(t) = \frac{\cos E(t) - e}{1 - e\cos E(t)}$$

The star-planet separation at any time *t* can then be determined from:

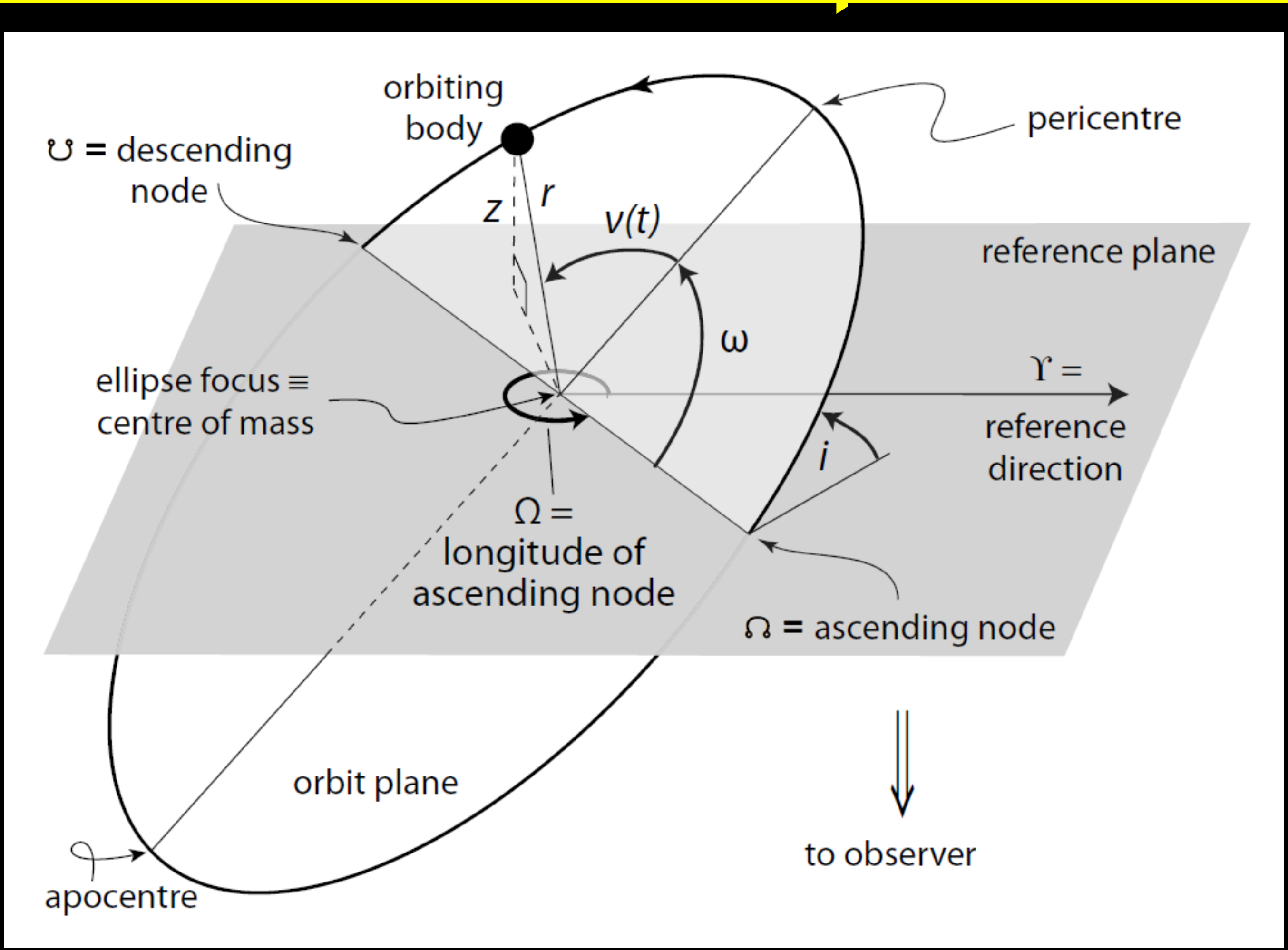
$$r = \frac{a(1-e^2)}{1 + e\cos f}$$

Always consider boundary conditions. e.g., *f* = 0 at periastron and *f* = π at apastron.

Time dependence of orbit

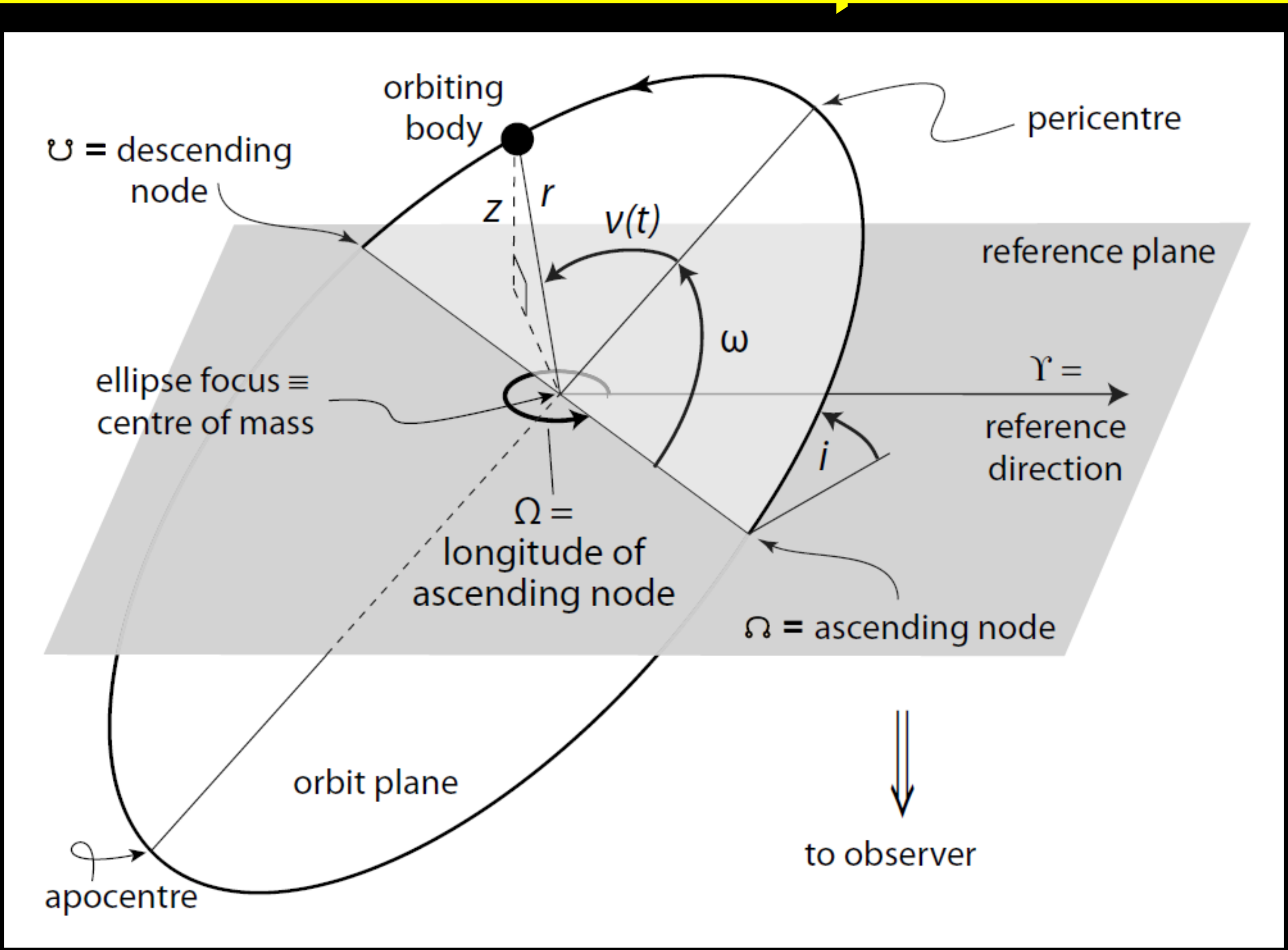
Example 3: What is the maximum orbital eccentricity a planet can have and still stay outside of the star?
Hint: separation must be $\geq R_* + R_p$

Orientation of orbit with respect to observer



There are several other angles we need to account for when including the direction at which the system is being observed.

Orientation of orbit with respect to observer

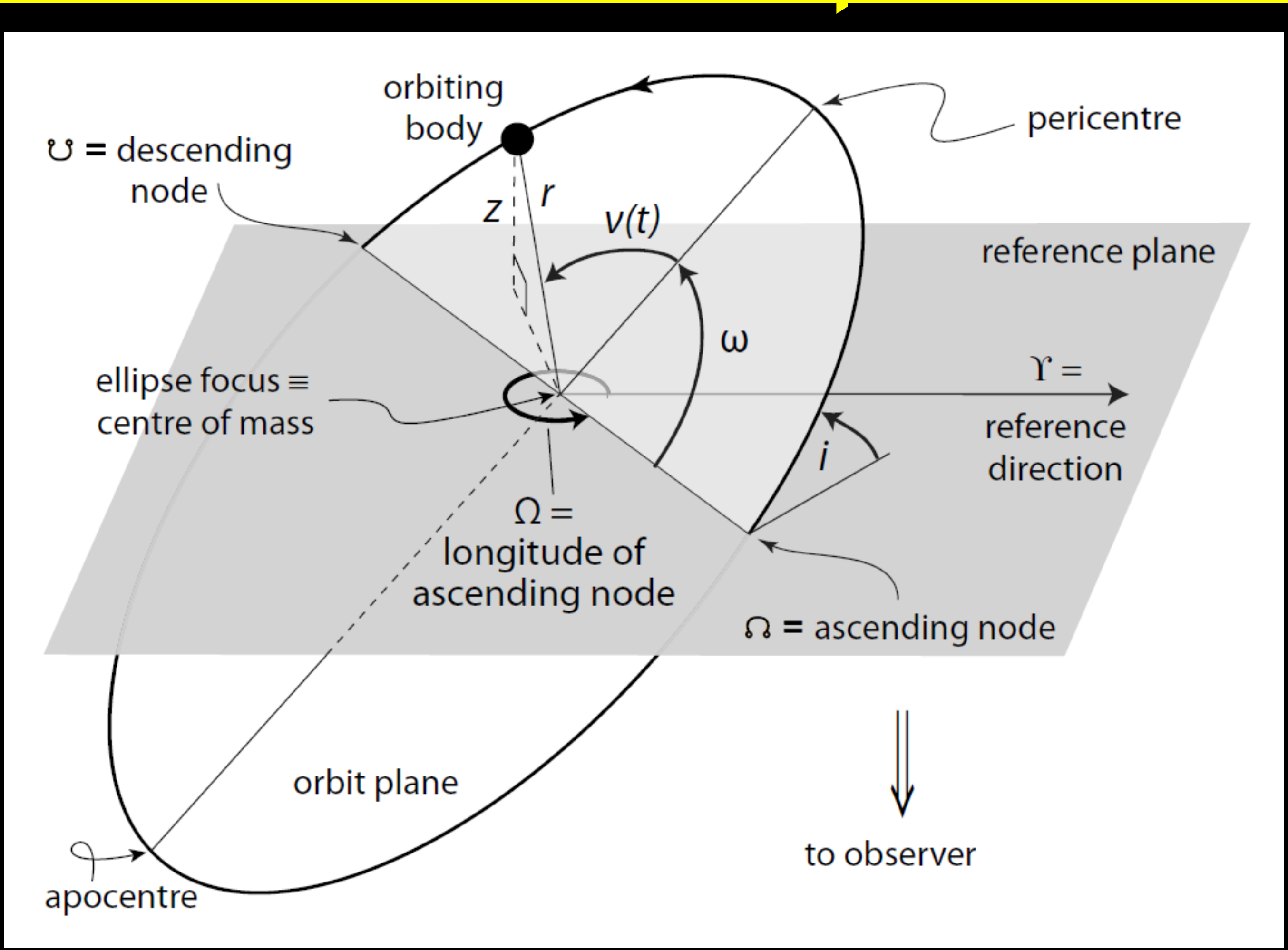


The first is the inclination of the orbit, *i*, with respect to the plane of the sky (reference plane).

For edge-on orbits, *i* = 90°. For face-on orbits, *i* = 0°.

The second is the argument of periastron, ω, which is the angle between periastron and the ascending node in the orbital plane.

Orientation of orbit with respect to observer



Note that some orbital elements are not normally used for exoplanets (such as longitude of the ascending node) since we usually don't know if the orbit is prograde or retrograde, or the inclination of the orbit.